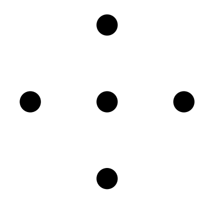
Gravitation Inside Earth

Let's start with the idea of *superposition*…

All the particles in the figure to the right have the same mass.
What is the force of gravity on the particle in the middle?



Answer: ZERO

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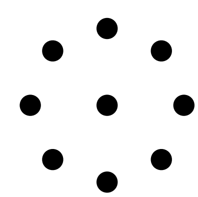

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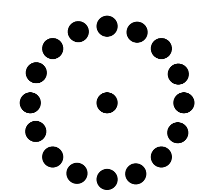
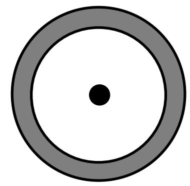


Answer: ZERO

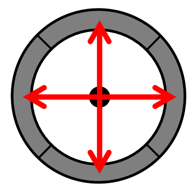
Yeah… The points on the outside always cancel each other out!

How about this one…

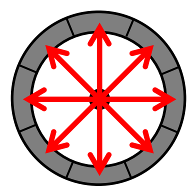
The ring of mass in the figure to the right is uniformly dense. What is the force of gravity on the particle in the middle?



Answer:
Check out the sketch of forces… Divide the ring into four equal areas, and the forces from each area cancel each other out.



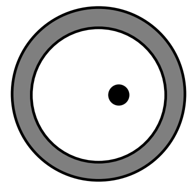
If we divide the ring into more equal areas, we still get the forces from each area canceling each other out.



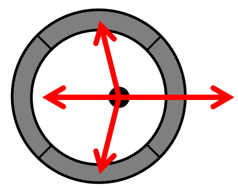
Yeah… A particle in the center of a ring of mass will have no net gravitational force on it due to the ring of mass!

How about this one…
Offset the mass inside the ring so it is no longer in the middle.

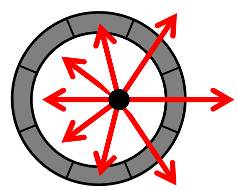
What is the force of gravity on the particle?



Answer:
Check out the sketch of forces… The closer areas of the ring will cause greater gravitational forces on the mass inside. The further areas of the ring will cause smaller gravitational forces on the mass inside. All the forces still cancel.



It works no matter how many areas the ring is divided into.



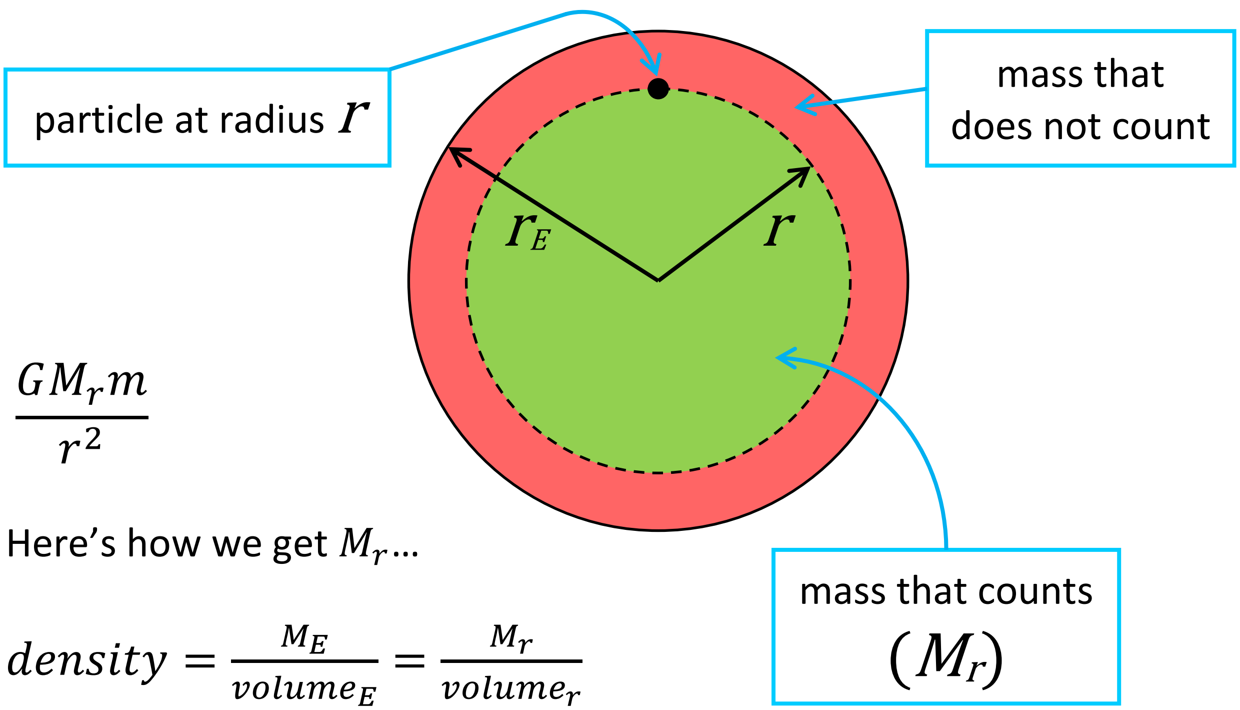
Yeah… A particle anywhere inside a ring of mass will have no net gravitational force on it due to the ring of mass!

Further, this works for a shell of mass…

A particle anywhere inside a uniformly dense shell of mass will have no net gravitational force on it due to the shell of mass!

A particle anywhere inside a uniformly dense shell of mass will have no net gravitational force on it due to the shell of mass!

The idea above is supremely important to finding gravity forces inside the earth. If a particle is anywhere between the surface and the center of the earth, any of earth's mass that is further from the center of earth than the particle does not count towards the force of gravity on the particle.



$$F_g = \frac{G M_r m}{r^2}$$

Here's how we get M_r...

$$density = \frac{M_E}{volume_E} = \frac{M_r}{volume_r}$$

$$M_r = \frac{M_{earth}}{volume_E} volume_r = \frac{M_E}{\frac{4}{3}\pi r_E^3}\left(\frac{4}{3}\pi r^3\right) = \frac{M_E}{r_E^3} r^3$$

$$F_g = \frac{G\left(\frac{M_E}{r_E^3} r^3\right) m}{r^2}$$

$$\boxed{F_g = G\left(\frac{M_E}{r_E^3}\right) m\, r}$$